Int. J. Heat Mass Transfer. Vol. 34, No. 8, pp. 2165–2166, 1991
Printed in Great Britain

0017–9310/91 \$3.00+0.00
Pergamon Press plc

TECHNICAL NOTES

A note on surface heat transfer coefficients

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(*Received for publication* 2 *October* 1990)

INTRODUCTION

PERHAPS the most common boundary condition which has been applied to heat conduction and diffusion problems equates the surface flux to a constant multiple of the difference between the surface temperature, or concentration, and the environmental value. The following analysis is devoted to heat conduction but the discussion can be transferred directly to any diffusion type problem.

For heat transfer problems the condition at a body surface, commonly referred to as the Newton cooling boundary condition, may be expressed as

$$-k\frac{\partial T}{\partial n} = H(T-T_\infty) \qquad (1)$$

where T is the surface temperature, T_∞ the environmental temperature, $\partial/\partial n$ the derivative in the direction of the outward normal, k the thermal conductivity of the body and H a constant dependent on environmental properties and conditions. The combination $h = H/k$ is called the heat transfer coefficient or surface conductance; an assumption of a constant value for h therefore presumes a constant value for H. Frequently the value of H is estimated from the expression $H = k'/\delta$ where k' is the thermal conductivity of the environment and δ the thickness of the layer across which a linear temperature profile would attain T_∞. The thickness δ is not the thermal boundary layer thickness unless the temperature fall is uniform but for any temperature variation into the environment it is possible to define the value of δ as illustrated in Fig. 1, which shows a typically thermal boundary layer profile which would be relevant to either a timewise development in a solid or stagnant fluid, or a steady state profile in streaming flow. The assumption of a constant heat transfer equates to having chosen a constant value for δ; this is certainly valid asymptotically (in time) for many problems where environmental conditions maintain a constant thermal boundary layer in the environment but it is well known that such an assumption is invalid for the initial period during which the thermal boundary grows towards this limit, though at any instant it is possible to calculate an effective thickness $\delta(t)$. For many problems the timespan is sufficiently great that this initial development is relevant during only a small portion of the phenomenon and the imposition of a constant heat transfer coefficient induces insignificant error, but there are cases in which the development of the environmental layer cannot be ignored. In such cases it becomes necessary to solve for the conduction process in both the body and the environment; this is, of course, always necessary if the environment is stagnant or happens to be a solid rather than fluid.

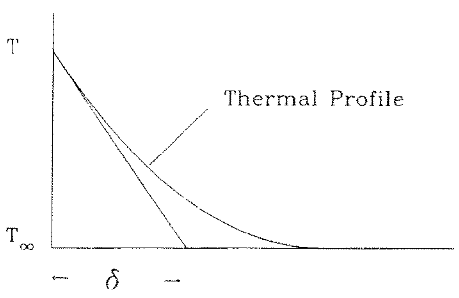

FIG. 1. A typical temperature profile in the environment and the associated layer thickness δ defined by Flux $= k'(T-T_\infty)/\delta$.

In this note a condition is constructed which links the flux to an integral involving the history of the surface temperature and thus enables the detailed calculation of the conduction in the environment to be avoided. This is deduced from a well-known exact solution for time-dependent one-dimensional conduction and provides a simple, but hitherto unspecified, surface condition which accommodates the timewise development of the surface layer.

ONE-DIMENSIONAL CONDUCTION

Consider conduction in a semi-infinite environment $\zeta \geqslant 0$ governed by

$$\frac{\partial T}{\partial t} = \kappa'\frac{\partial^2 T}{\partial \zeta^2} \qquad (2)$$

where κ' is the thermal diffusivity. If the region is initially at uniform temperature T_∞ and the temperature at $\zeta = 0$ for $t \geqslant 0$ is specified as $\hat{T}(t)$ the temperature distribution is given (see p. 63 of Carslaw and Jaeger [1]) by

$$T(\zeta,t) = \frac{\zeta}{2(\pi\kappa')^{1/2}}\int_0^t (\hat{T}(\lambda)-T_\infty) \times \frac{\exp\{-\zeta^2/4\kappa'(t-\lambda)\}}{(t-\lambda)^{3/2}}\,\mathrm{d}\lambda \qquad (3)$$

and on differentiating this result, with respect to ζ, it is readily found that

$$\left(\frac{\partial T}{\partial \zeta}\right)_{\zeta=0} = -\frac{1}{(\pi\kappa')^{1/2}}\frac{\mathrm{d}}{\mathrm{d}t}\left(\int_0^t \frac{(\hat{T}(\lambda)-T_\infty)}{(t-\lambda)^{1/2}}\,\mathrm{d}\lambda\right). \qquad (4)$$

From this it is possible to calculate the surface flux knowing only the time history at the surface, or vice versa; note that the temperature gradient at $\zeta = 0$ is directed into the environment. Alternatively, on equating heat flux from the body to that into the environment, we may derive the relationship

$$k\frac{\partial T}{\partial n} = -\frac{k'}{(\pi\kappa')^{1/2}}\frac{\mathrm{d}}{\mathrm{d}t}\left(\int_0^t \frac{(\hat{T}(\lambda)-T_\infty)}{(t-\lambda)^{1/2}}\,\mathrm{d}\lambda\right) \qquad (5)$$

linking the surface temperature gradient within the body and its surface temperature. Since this must be satisfied throughout the conduction process it can be applied as a surface boundary condition for the conduction in the body without further reference to the environment.

Alternatively it is possible to define a 'transient heat transfer coefficient' $\hat{h}(t)$, based on the temperature at $\zeta = 0$, by

$$\hat{h}(t) = \frac{1}{(\pi\kappa')^{1/2}}\frac{(k'/k)}{(\hat{T}(t)-T_\infty)}\frac{\mathrm{d}}{\mathrm{d}t}\left(\int_0^t \frac{(\hat{T}(\lambda)-T_\infty)}{(t-\lambda)^{1/2}}\mathrm{d}\lambda\right) \tag{6}$$

and transient functions $\hat{H}(t) = k\hat{h}(t)$ and $\hat{\delta}(t) = k'/(\hat{h}(t))$. The last of these is particularly useful in assessing a period in which it is more appropriate to use this model than that of a constant (asymptotic) thickness. Generally the variable heat transfer coefficient would be applied until the value of $\hat{\delta}(t)$ attains the value of δ. Also the time varying heat transfer coefficient will be appropriate for all time in problems in which there is insufficient fluid motion to restrict the growth of the boundary layer.

RESULTS AND DISCUSSION

In the previous section attention has been focused on one-dimensional conduction. Clearly it is not precise to use equation (6) as a formula for calculating the transient heat transfer coefficient for non-planar boundaries, but for cases in which the radius of curvature of the surface is greater than the thickness of the developing thermal boundary layer in the environment any errors will be of the second-order effect; the validity of this constraint can easily be checked as time lapses. In this respect it should be recognized that the formula for H, and hence h, also needs amending from $H = k'/\delta$ to accommodate curvature. It does not seem necessary to present a lengthy discussion section because the application to one-dimensional problems is so simple that the significance of the result may be assessed with comparative ease. As has been mentioned above it is always incorrect to impose a constant value for the heat transfer coefficient if the environment is a solid or if the fluid is stagnant. In order to see the model can be equally relevant in problems where there is an asymptotic film thickness consider the idealized situation in which the surface temperature increases linearly with time. In the situation where $\hat{T}(t)-T_\infty = \beta t$ substitution in equation (6) yields

$$\hat{h}(t) = \frac{2k'}{k(\pi\kappa' t)^{1/2}}$$

and

$$\hat{\delta}(t) = \tfrac{1}{2}(\pi\kappa' t)^{1/2}.$$

The result is independent of β but that is not surprising because this is a well-known solution of the heat conduction problem in a semi-infinite region. It is extremely obvious, but nevertheless important to recognize that the form of $\hat{\delta}(t)$ predicts there is a period of time in which the time varying thickness $\hat{\delta}(t)$ is less than δ and hence the transient heat transfer coefficient will be greater than h, moreover the period will be greater for poorly conducting, or diffusing, environments. Two examples in which there is need to use equation (6) rather than a constant h are the heat conduction aspects of laser soldering and certain diffusion phenomena concerning submerged plant aeration.

Laser soldering is a technique which has considerable importance in fixing microcomputer components because it permits large amounts of heat to be directed at small areas in a small time interval and avoid excessive (and damaging) global heating. Typically the solder paste, which has a surface exposed to the air (or perhaps nitrogen) is raised to approximately 600 °C in 0.2 s [2]. On the assumption that the surface temperature rise is linear it follows that the heating phase could be complete before the developing thickness $\hat{\delta}(t)$ attained the sort of value frequently imposed for δ; typically $\delta \in [0.001\ \mathrm{cm}, 0.1\ \mathrm{cm}]$ (see p. 20 of Carslaw and Jaeger [1]).

However, it is not just rapid processes which demand caution. Consider, for example, diffusion of carbon dioxide from submerged plants to water. In this case the relevant diffusivity D', which replaces both k' and κ' in the heat transfer theory, has the value $1.86 \times 10^{-5}\ \mathrm{cm^2\ s^{-1}}$ and it is therefore possible for appreciable time (up to the order of an hour) to lapse before the asymptotic condition relevant to stirring becomes a good boundary condition [3].

CONCLUSION

It has been shown that for conduction problems in which there is heat flow into a uniform unbounded conducting environment the detailed calculation throughout the environment can be replaced by a surface condition which relates the surface temperature and surface flux.

The practical use of the boundary condition is envisaged in numerical calculations of heat transfer or diffusion problems. In each case the accumulation of the time-dependent integral included in equation (6) presents only a minor computational complication.

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